STUDY OF SINS WORK IN THE CONDITIONS OF HIGH LATITUDES TAKING INTO ACCOUNT THE ERRORS OF REAL SENSORS *

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Abstract

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The article proposes an algorithm for the functioning of SINS based on switchable algorithms in the Euler-Krylov angles and quaternions for work on mobile objects at high latitudes and under high values of pitch angle. Variants of the object movement through the north pole as well as execution of dead-loop are considered. The simulation results of the work of ideal system as well as system with taking into account the real sensor errors are given.

The problem

The use of the classical orientation parameters like Euler-Krylov angles, for example, for the SINS work is inspired by the presence of such advantages as visual geometric interpretation and simplicity of the differential equations constructed on their basis, which specifies reduction of the onboard computer computational load as well as high accuracy of the orientation and navigation problem solution. However equations based on Euler-Krylov angles have disadvantages due to the limited range of the measured angles. Kinematic equations degenerate with the specific values of pitch angle, which leads to the failure of the navigation system. The same problems appear with the SINS work at high latitudes. We need to note another method described in [1] wherein the gyroazimuths reoriented to a new conventional pole, which was positioned at the geographic equator, when the system traversed 80th parallel.

Task solution

The work of SINS based on the kinematic equations in Euler-Krylov angles has a set of limitations connected to the range of measured angles since the system has the singular points (with the pitch of ±90° and the latitude of 87°<φ, -87°>φ) when the kinematic equations in Euler-Krylov angles are degenerated, which follows, for example, from [2], [3].

$$\dot{\gamma} = \left[\left(\omega_{\eta_1} + \omega^k_{\eta_1}\right)\cos\psi - \left(\omega_{\eta_3} + \omega^k_{\eta_3}\right)\sin\psi\right]\cos^{-1}\theta; \tag{1}$$

$$\dot{\lambda} = -\frac{\omega^k_{\zeta 1}}{\cos\varphi} - U, \tag{2}$$

where φ, λ are estimations of geographic latitude and longitude angles; ψ, θ, γ are estimations of orientation angles; $\omega_{\eta_1}, \omega_{\eta_3}$ are mobile object absolute angular rates that projected on axes of a free in azimuth reference frame; $\omega^k_{\eta_1}, \omega^k_{\eta_3}$ are correction angular rates along a free in azimuth horizon reference frame; $\omega^k_{\zeta 1}$ is correction angular rates along the geographic reference frame [3], U is angular rate of the Earth.

At the same time the differential equations in Euler-Krylov angles give the higher accuracy that take place with the small values of pitch angles and under the condition of low latitudes. At the same time the quaternion algorithms are free of singular points (they do not degenerate at any position of mobile object in the space) and practically do not contain trigonometric functions. Therefore, it is appropriate to use the switching between the algorithms in the Euler-Krylov angles and quaternions in order to solve the orientation and navigation problem for mobile object which moves at the high latitudes, or executes complex maneuvers by the pitch angle.

The correcting terms of the position-integral horizontal correction from the signals of accelerometers are included into the kinematic equations which referenced to the free in the azimuth horizon trihedron. The radial-position

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horizontal correction mode is used under the initial alignment, and only the integral correction with Schuler's period is used under working mode condition. The conditions of switching of algorithms from the differential kinematic Euler equations to the quaternion ones and vice versa are provided.

In order to validate mentioned conditions the special movement modes of mobile object along with the usual movement mode are considered. Such modes are the passage of north and southern poles as well as performing the dead loop by the mobile object. Under the given conditions the system switches to the calculation of quaternions with the capability of their conversion to Euler-Krylov angles:

$$\psi = arctg\left(\frac{2(v_0v_2 - v_1v_3)}{v_0^2 + v_1^2 - v_2^2 - v_3^2}\right),\ \theta = arcsin(2(v_1v_2 + v_0v_3)),\quad \gamma = arctg\left(\frac{2(v_0v_1 - v_2v_3)}{v_0^2 + v_2^2 - v_1^2 - v_3^2}\right); \qquad (3)$$

$$\varphi = arcsin(2(\varepsilon_1\varepsilon_2 - \varepsilon_0\varepsilon_3));\ \lambda = arcsin(-2(\varepsilon_2\varepsilon_3 - \varepsilon_0\varepsilon_1)) - Ut,$$

where v_0 to v_3 are orientation quaternions: ε_0 to ε_3 are navigation quaternions.

The method described in [2] which takes into account the errors of the real sensors is used in order to ensure the adequacy of the real BINS work mathematical simulation. Verification of the efficiency of proposed switching scheme was carried out by the mathematical simulation using the algorithms of ideal system working, i.e. without the errors of sensors. In order to do this the following cases of mobile object motion were considered. In the first case the mobile object was underwent the effect of tossing by the course, pitching and rolling and moved northwards as long as it overpassed the pole. At the latitude of 87° algorithms in form of Euler equations [3,4] were switched to quaternion ones [5]. And after mobile object runs out the high latitudes the algorithms switched back. Additionally, in order to reduce the errors of pitch angle determination in quaternion algorithms the switching of the vertical correction at the latitudes beyond 89° is specified in the following form:

$$\omega^k_{\zeta_2} = \begin{cases} \omega^k_{\zeta_1}\dfrac{2(\varepsilon_1\varepsilon_2 - \varepsilon_0\varepsilon_3)}{\sqrt{1 - 4(\varepsilon_1\varepsilon_2 - \varepsilon_0\varepsilon_3)^2}}, \varphi \le 89^o; \\ -2U(\varepsilon_1\varepsilon_2 - \varepsilon_0\varepsilon_3) - \dfrac{v_{\zeta_3}}{R}\dfrac{2(\varepsilon_1\varepsilon_2 - \varepsilon_0\varepsilon_3)}{\sqrt{1 - 4(\varepsilon_1\varepsilon_2 - \varepsilon_0\varepsilon_3)^2}}, \varphi > 89^o, \end{cases} \qquad (4)$$

where $\omega^k_{\zeta 2}$ is vertical correction angular rates along the geographic reference frame, v_{ζ_3} is estimation of the eastern component of mobile object relative velocity (we considered the case when v_{ζ_3} was equal 0), R is radius of the Earth. In the second case the mobile object performed evolution by the pitch from 0° to 360°, thus its trajectory had form of dead loop. The switching to quaternion algorithms is issued when the pitch angle is greater than ±30°.

The Runge-Kutta numerical integration method of the order 4 (5) [6] with integration step that automatically varied with the maximum value of 0.01 sec is used under the mathematical simulation. A compensation of Coriolis acceleration and velocity error of the gyrocompass mode is applied. The Earth model was adopted in form of sphere. During the beginning of simulation (t <500 sec) the initial alignment is made on static relatively the Earth object (see fig. 1). After that from the time moment t=1000 sec and till the end of simulation the course Ψ, pitch θ and roll γ tossing is turned on. Also uniformly accelerated motion along ζ_1 axis is assigned for mobile object. Whereupon the mobile object is moved at a constant speed 1000 km/hour. SINS working time is $1.4 \cdot 10^4$ s.

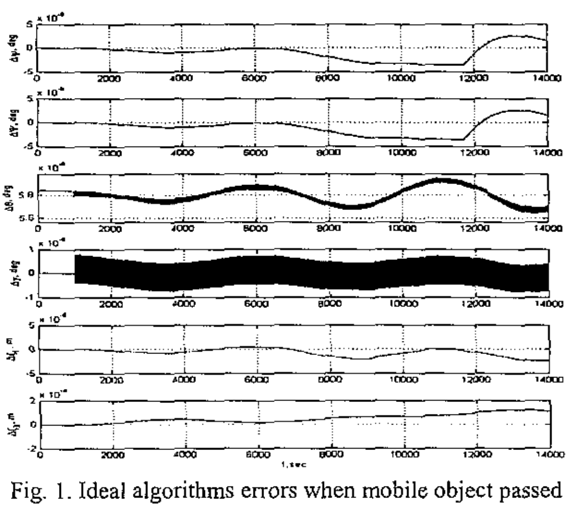

Fig. 1. Ideal algorithms errors when mobile object passed the north pole

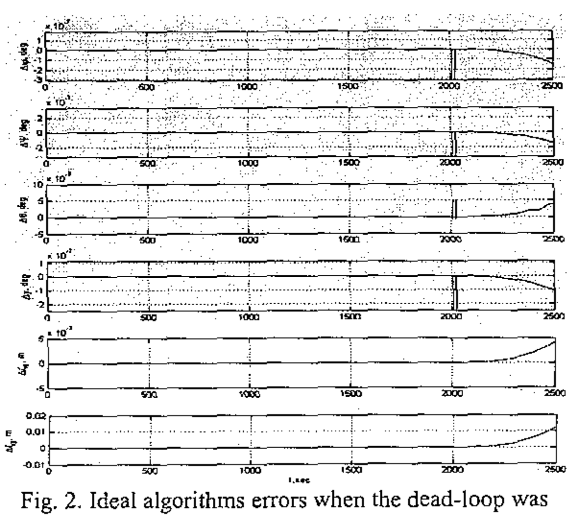

Fig. 2. Ideal algorithms errors when the dead-loop was executed

Fig. 1 shows the diagram of the orientation angles determination errors as well as coordinates determination errors of the SINS working under ideal algorithms. It is clear from the diagrams that the errors of the pitch and roll angles were slightly accumulated. And by the moment of time t=1.4·10^4 s they reached values of $\Delta\theta$=6·10^{-9} °; $\Delta\gamma$=8·10^{-10} °; increase of course error was greater and comprised value of $\Delta\Psi$=2.5·10^{-9} °. The diagram shows the moment of time when the vertical correction was switched (about 10^4 s). The errors of determination of coordinates at the moment of movement end were $\Delta\zeta_1$=2·10^{-5} m; $\Delta\zeta_3$=1.2·10^{-4} m. When the mobile object executes the dead-loop (see fig. 2) the errors form next values. The errors of pitch and roll angles are $\Delta\theta$=0.4·10^{-7} °; $\Delta\gamma$=1.1·10^{-7} °; the course angle error is $\Delta\Psi$=1.5·10^{-7} °; the coordinates determination errors are $\Delta\zeta_1$=4·10^{-3} m; $\Delta\zeta_3$=1.2·10^{-2} m.

The mathematical simulation of functioning of SINS based on the real sensors is carried out with taking into account the errors in form of gyro drift rates, accelerometer biases, scalefactor deviations, its nonlinearity and asymmetry as well as output signal noises. The values of sensor errors are determined on base of records of the real devices (FOG SRS-2000 by LPC «Optolink» Ltd and accelerometers AKP-2 by SCAP). In order to obtain the sensor noise characteristics the sensor signals were processed and the Allan variance factors were calculated (N, B, K, q_c, t_c) (see fig. 3). Then these parameters were used in procedure which formed the noise errors of signals of SINS sensors under the mathematical simulation (see fig. 4, here the synthesized and the real noises are practically equivalent).

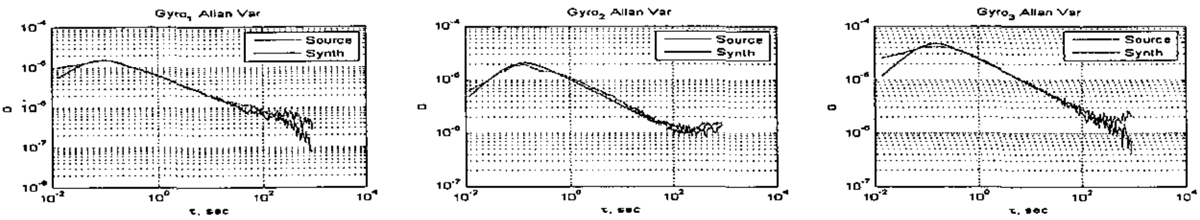

Fig. 3. Diagram of Allan variance of the gyro source and synthesized signals

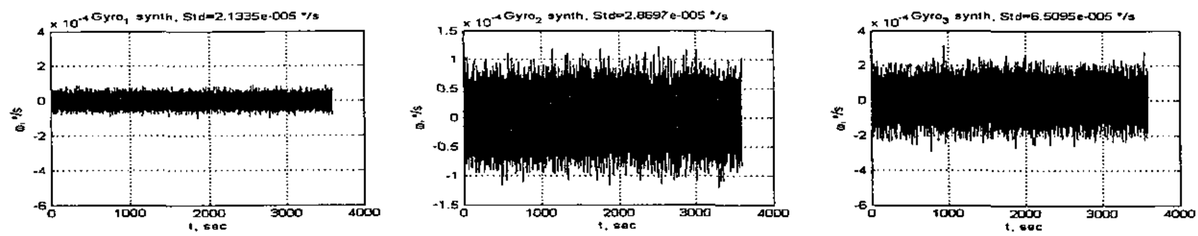

Fig. 4. Diagrams of the gyro synthesized signals

The following errors of sensor were taken into account: systematic components of the FOG drift rates, $\Delta\omega_{xi}$=0.005 (i=1,2,3) °/hour; FOG scale-factor errors, $\delta\omega_{xi}$=0.8·10^{-4} (i=1,2,3); asymmetry errors of static characteristics, $A\omega_{xi}$=0.8·10^{-6} (i=1,2,3); accelerometer bias, ΔW_{xi}=4·10^{-4} m/s^2; accelerometer scale-factor errors, δW_{xi}=10^{-4} (i=1,2,3).

Figure 5 shows the results of the SINS mathematical simulation under given conditions when the mobile object passed the north pole. At the moment of time t=1.4·10^4 sec the errors take next values: $\Delta\psi$=0.75 °; $\Delta\Psi$=0.75 °; $\Delta\theta$=1.3·10^{-2} °; $\Delta\gamma$= 4·10^{-2} °; $\Delta\zeta_1$=2.1 km; $\Delta\zeta_3$=16.2 km.

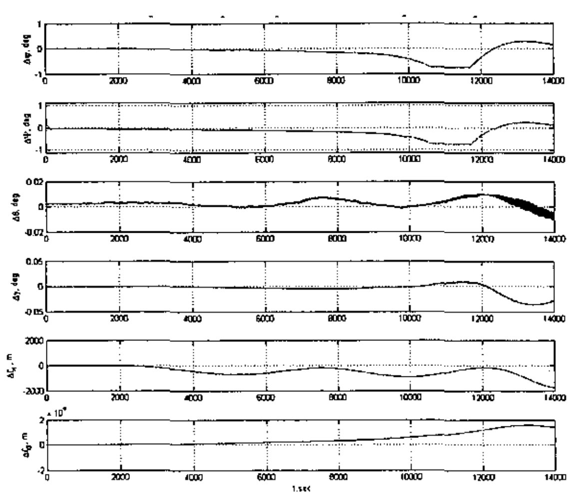

Fig. 5. SINS errors with taking into account the errors of sensors when mobile object passed the north pole

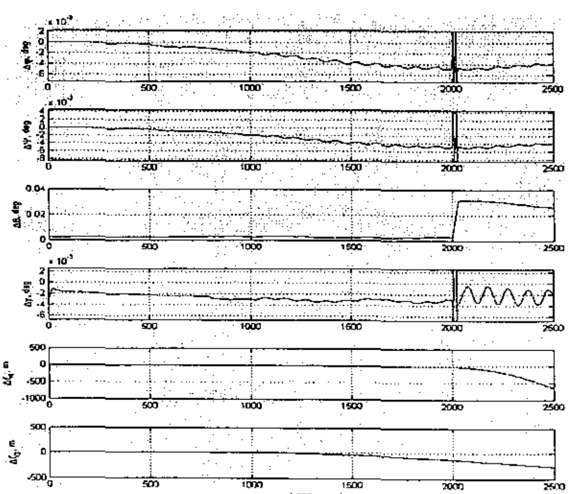

Fig. 6. SINS errors with taking into account the errors of sensors when the dead-loop was executed

Under the similar conditions when mobile object executed dead-loop the errors take next values: $\Delta\psi=5\cdot10^{-3}$ °; $\Delta\Psi=5\cdot10^{-3}$ °; $\Delta\theta=3.2\cdot10^{-2}$ °; $\Delta\gamma=4\cdot10^{-3}$ °; $\Delta\zeta_1=0.63$ km; $\Delta\zeta_3=0.3$ km for 2500 sec (the execution time of the very dead-loop is 30 sec).

The results of the simulation show that system accuracy remains at comprehensible level when the switching of the algorithms of calculation of orientation parameters from differential kinematic Euler equations to quaternion ones was used and errors of sensors were taken into account. This fact confirms a suitability of the complex algorithm application.

Conclusion

It is shown that the algorithms provided the switching from differential kinematic Euler equations with correction terms to quaternion ones have the admissible accuracy when working in absence of errors of sensors as well as in the presence of the sensor errors determined using experimental records. The Allan variance is used for estimation of the noise parameters of signals of sensors (FOG SRS-2000 and accelerometers AKP-2). Numerically the values of the errors of sensors were: FOG drift rate was $\Delta\omega_{xi}=0.005$ ($i=1,2,3$) °/hour, scale-factor error was $\delta\omega_{xi}=8\cdot10^{-5}$ ($i=1,2,3$), asymmetry of the FOG static characteristics was $A\omega_{xi}=0,8\cdot10^{-6}$ ($i=1,2,3$), gyro output noise standard deviation was $2.3\cdot10^{-5}$ to $6.6\cdot10^{-5}$ °/s, accelerometer bias was $\Delta W_{xi}=4\cdot10^{-4}$ m/s^2, accelerometers scale-factor error was $\delta W_{xi}=10^{-4}$ ($i=1,2,3$), standard deviation of output noises of accelerometers was $2.3\cdot10^{-4}$ to $6.1\cdot10^{-4}$ m/s^2.

With taking into account the real errors of sensors the following results were received. In the case of mobile object passing the north pole the estimations of autonomous SINS coordinate determination errors were $\Delta\zeta_1=2,1$ km for north direction and $\Delta\zeta_3=16.2$ km for east direction as well as the error of the course determination was $\Delta\Psi=0.75°$ for the time of $t=1.4\cdot10^4$ sec. In the case of mobile object dead-loop performing the coordinate determination errors were $\Delta\zeta_1=0.63$ km for north direction and $\Delta\zeta_3=0.3$ km for east direction as well as the error of the course determination was $\Delta\Psi=5\cdot10^{-3}$ ° for the time of $t=2.5\cdot10^3$ s.

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