# EXPERIMENTAL RESEARCH OF FOG AND ACCELEROMETERS AND THE ANALYSIS OF THEIR PARAMETERS FOR SINS ERROR PREDICTION

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## Abstract

**Key words:** strapdown inertial navigation system, navigation, errors, course, noises, Allan variance

*The errors of fiber-optic gyroscopes (FOG) and accelerometers, such as zero drift and zero bias, asymmetry and intrinsic noises, are analyzed as the result of laboratory tests. Allan variation is used in the analysis of noise influence. The approach to error prediction of the strapdown inertial navigation system (SINS) output parameters taking into account the errors of FOGs and quartz pendulum accelerometers is described. This approach is based on the mathematical simulation of SINS operation. The diagrams of SINS errors versus the main errors of gyroscopes and accelerometers are obtained.*

## The problem

The natural method of determining SINS errors is experimental while the investigated system is mounted into the object of application along with standard SINS or INS. So the motion parameters determination processes using FOG and accelerometer signals and, above all, the coordinates and orientation parameters of the object are recorded by both systems simultaneously. During information post processing the errors of the investigated system and the sensors are easy to determine. In practice this possibility is presented exceptionally rare. It is more frequently that a single SINS, sometimes a single inertial unit and even a single sensor, are tested on the mobile object (MO). In the case of a single SINS, the errors of navigation parameters can be determined by means of the collation of the SINS signals with GPS/GLONASS signals, but practically the errors of orientation parameters as well as the errors of FOG and accelerometers cannot be estimated.

Therefore the analytic estimations as well as mathematical simulation methods are used for prediction of SINS errors. Usually the signals of FOG and accelerometers (further referred to as sensors) are available. Often the records are made in laboratory or plant conditions and usually contain intrinsic noises of sensors as well as other errors. As regards the MO movement, the type of the MO and the parameters of its rolling, speed and acceleration, including linear motion, are commonly known. In some cases the records from single sensors, the motion parameters of similar type objects are available. In the mathematical simulation a complete system of differential equations and finite equations for MO motion and SINS require a complete set of linear and angular motion parameters of the object, its angular and linear vibration at the SINS placement location and errors of sensors, etc. It is reasonable that some of MO motion parameters and some of SINS parameters must be defined.

FOG zero drift and accelerometers zero bias, scale factor errors and its nonlinearity and asymmetry are set using statistical processing of the test results of the sensors. The intrinsic noises of the sensors are recorded during tests, and then Allan variance coefficients are determined. Also noises of the sensors are analyzed by means of the spectral and correlation techniques. Then "color" noises are formed from the "white" noise using the obtained results. The mathematical simulation of SINS is made of 10 to 15 cycles of 7–20 thousand seconds each. At that MO motion is given by analogy with other objects in the form of kinematic relations and differential equations.

During subsequent statistical processing of SINS mathematical simulation results the errors are averaged from the above mentioned number of operation cycles and operation duration. Standard deviations and other characteristics are determined. Ways to reduce errors of the SINS are planned from the analysis of the obtained results.

## Task solution

Ship is taken as a mobile object. Ship is under average rolling and moves at speeds of up to 40 knots. The SINS consisting of three-axis FOG and three-axis specific force meter based on quartz compensatory accelerometers with

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onboard computer. Integration step is up to 0,01-0005 sec. Gyroscope errors parameters were varied. Accelerometers errors were relatively smaller and were fixed along the zeros bias, Allan variation parameters and other characteristics. Sensors errors values are determined by experiments. During results processing the Allan variation coefficients (*B*, *N*, *K* etc) were obtained and power spectrum noise density diagrams were plotted. After that the "color" noises generation is made from "white" noise by the computer program using developed technique with taking in account Allan Variance coefficients. Coefficients were changed and autonomous SINS work cycles were simulated. In addition to the navigation parameters, the orientation parameters including yaw angle and course angle that measured continuously using gyrocompass equations were taken out.

The power spectrum noise densities of various gyroscopes were compared. Published data for SRS-1000 by LPC "Optolink" Ltd, VG-951 and VG-910FOS by STC "Fozoptika", VOG-06/100 by PNPPK were used. The Allan variance is calculated for SRS-1000, VG-951. "Color" noises of VG-951 and VOG-06/100 were generated by developed procedures using available data. SRS-1000 was experimentally researched and the Allan variance coefficients were calculated. Models of "color" noise were constructed. As well as a zero bias and scale factor errors were determined.

Further the "color" noises synthesis results for one component of the FOG signal are given. Figure 1 presents the recorded signal of "X" channel of device BGD which is stationary relatively to the Earth and the Allan variance of this signal. Similarly Figure 2 shows the recorded signal of FOG VG-951 and its Allan variance.

Figures 3 and 4 present signal synthesized in accordance with [9,10,11,12] and the Allan variance diagram. It is not difficult to see that they are identical, respectively. Similar operations are made for other channels.

Analogously the signals of the accelerometers were synthesized. Figure 5 shows accelerometer AKP-2 signal and Allan variation diagram.

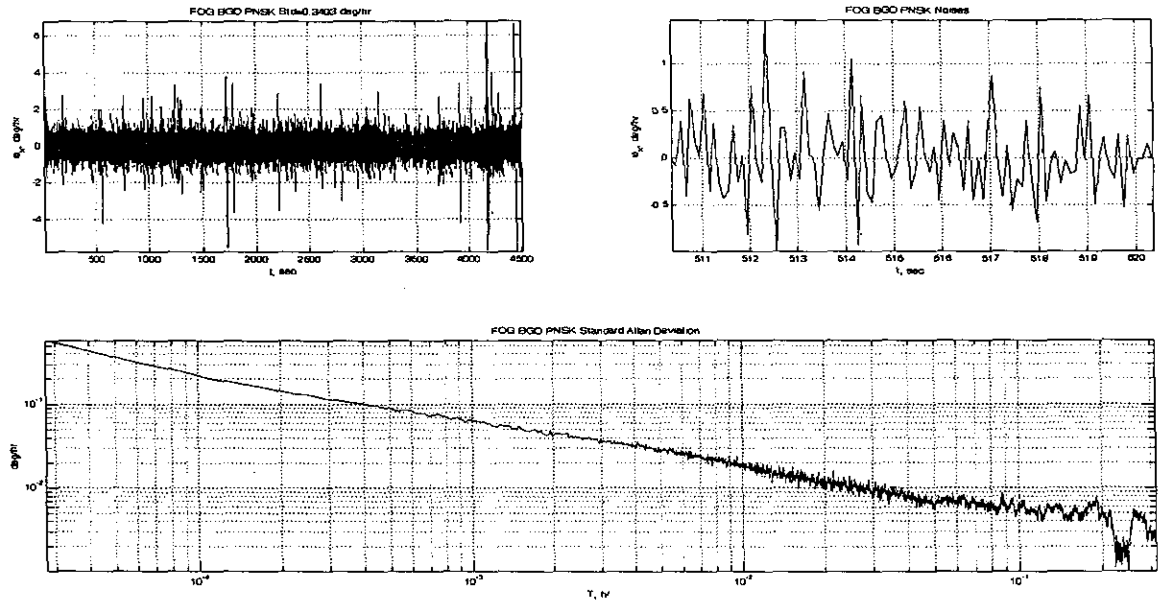

Fig. 1. PNSK 40-018 signal and Allan variance diagram

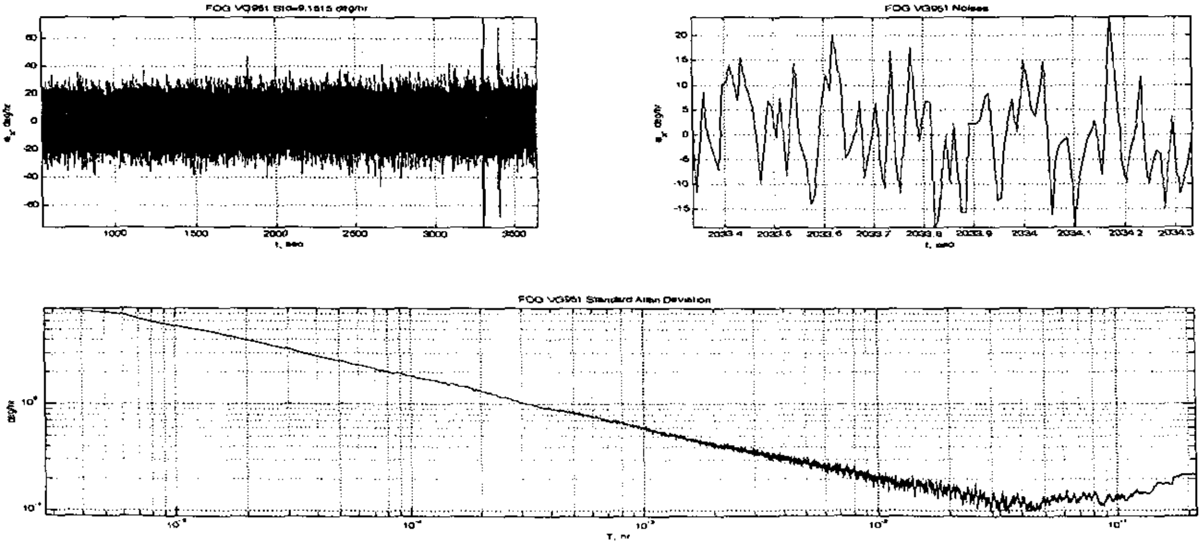

Fig. 2. VG-951 signal and Allan variance diagram

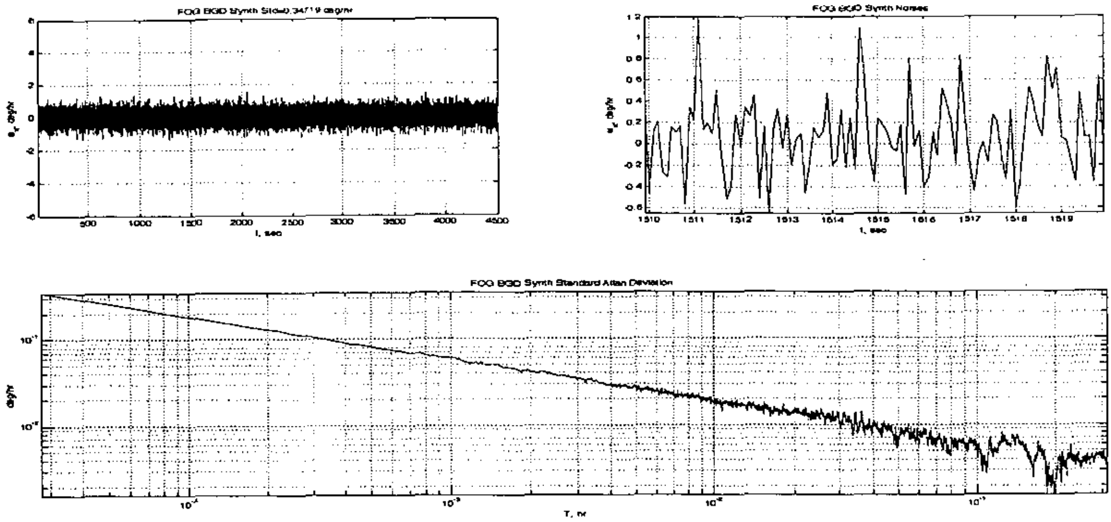

Fig. 3. PNSK 40-018 synthesized signal and Allan variance diagram

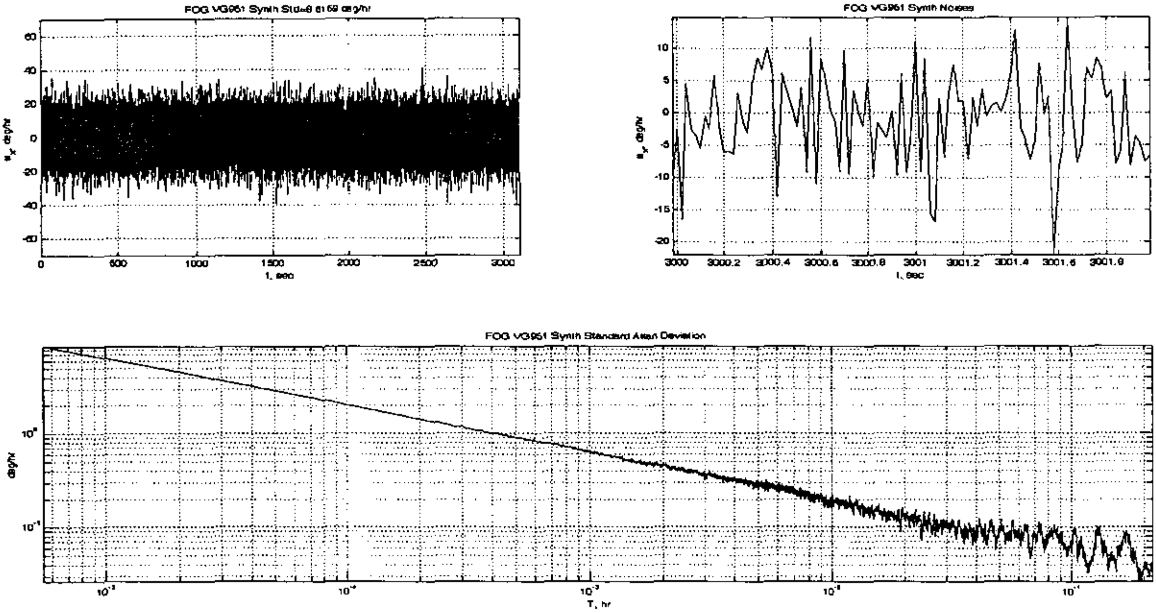

Fig.4. VG-951 synthesized signal and Allan variance diagram

Taking into account the mobile object movement parameters, sensors parameters, "color" noises synthesized on base of the calculated Allan variance coefficients and other sensors errors the mathematical modeling of the SINS was made using a system of differential equations. As a result SINS error were determined. It allows to compare an influence of the sensors errors level under the same conditions and to estimate an influence of FOG noises on the SINS accuracy.

Orientation algorithms and movement parameters which are formed on base of [5] and supplemented by course angle determination algorithms [12] as well as by navigation parameters determination algorithms by the formulas (1), (2) according to [6.7] were used for SINS work mathematical simulation.

$$\dot{\hat{\varphi}} = \omega^k_{\zeta 3};\ \dot{\hat{\lambda}} = -\frac{\omega^k_{\zeta 1}}{\cos\hat{\varphi}} - U; \tag{1}$$

$\hat{\varphi}, \hat{\lambda}$ are object latitude and longitude estimations, $U$ is Earth angular rate, $\omega^k_{\zeta 3}$ is angular correction rate in frame $0\zeta_1\zeta_2\zeta_3$.

$$\frac{d\hat{\zeta}_1}{dt} = R\dot{\hat{\varphi}};\ \frac{d\hat{\zeta}_3}{dt} = -R\dot{\hat{\lambda}}\cos\hat{\varphi}; \qquad )$$

$\hat{\zeta}_1, \hat{\zeta}_3$ are estimations of mobile object position coordinates, $R$ is Earth radius.

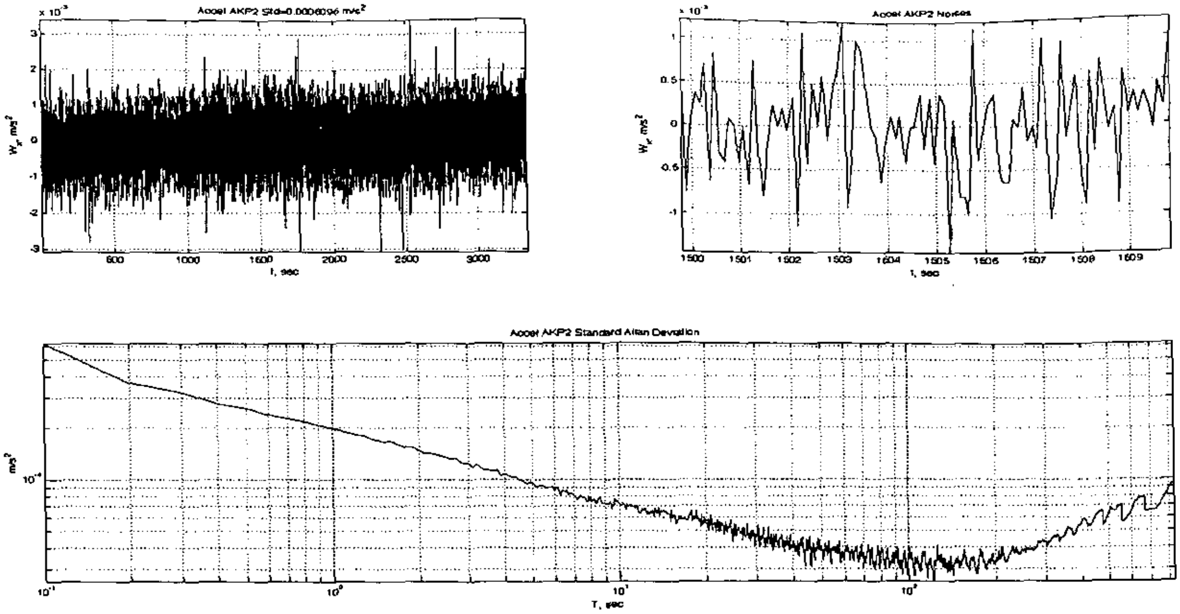

Fig. 5. Accelerometer AKP-2 signal and Allan variance diagram

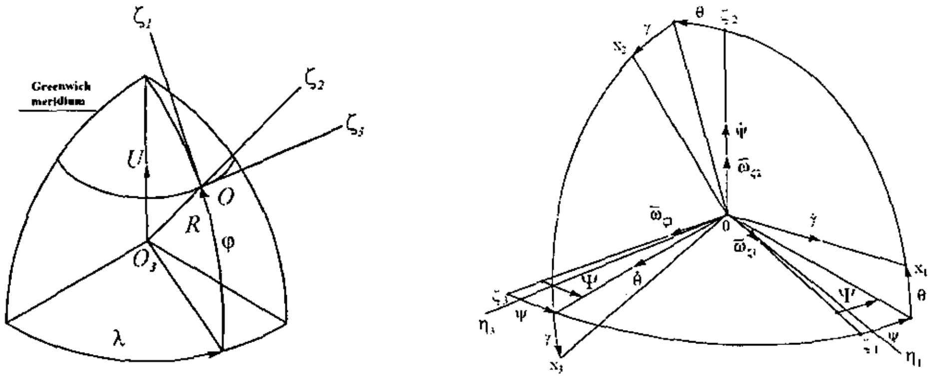

Fig. 6. Used coordinate frames

The simulation programs is based on the Runge-Kutta numerical method of the order 4 (5) [3] with integration step which is automatically varied with the maximum value of 0.01 sec or less. Compensation of Coriolis acceleration and velocity error of the gyrocompass mode is applied. To evaluate the effectiveness of algorithms of the functioning process the mathematical simulation using the ideal work algorithms was done. It implies that the sensor errors aren't taken into account and accurate algorithms were replaced by discrete ones using the Runge-Kutta scheme. The Earth model was adopted in form of sphere.

During simulation the initial alignment is made on the static relatively Earth object in the beginning ($t$ <500 sec) [5]. Then at time $t$=1000 sec and thru the end of simulation the course, pitch and roll tossing is turned on.

$$\Psi = \Psi_a \, sin(\omega_\Psi t + \Delta\chi_\Psi)\,; \quad \theta = \theta_a \, sin(\omega_\theta t + \Delta\chi_\theta)\,; \quad \gamma = \gamma_a \, sin(\omega_\gamma t + \Delta\chi_\gamma) \tag{3}$$

$\theta_a$, $\gamma_a$, $\Psi_a$ are the amplitudes of pitch, roll, and of course tossing which are changed smoothly by the exponential law:

$$\theta_a = 0{,}1 \cdot (1 - e^{-0{,}6(t-t_0)})\,; \Psi_a = 0{,}1 \cdot (1 - e^{-0{,}6(t-t_0)})\,; \gamma_a = 0{,}1 \cdot (1 - e^{-0{,}6(t-t_0)}) \tag{4}$$

Here $\omega_\Psi$, $\omega_\theta$, $\omega_\gamma$ are the course, pitch and roll tossing rates equal respectively 0,628 rad/sec, 0,314 rad/sec and 1,256 rad/sec, $\Delta\chi_\Psi$, $\Delta\chi_\theta$, $\Delta\chi_\gamma$ are the course, pitch and roll tossing phases equal respectively 0 rad, 0,7 rad and 0,4 rad. Also uniformly accelerated motion in two axes $\zeta_1$ and $\zeta_3$ is assigned for mobile object. Whereupon the mobile object is moved at a constant speed:

$$t\text{=1200...1220 c,} \quad v_{\zeta 1} = \frac{dv_{\zeta 1}}{dt}(t-1200) \text{ m/s;} \quad v_{\zeta 3} = \frac{dv_{\zeta 3}}{dt}(t-1200) \text{ m/s;} \quad \frac{dv_{\zeta 1}}{dt} = \frac{dv_{\zeta 3}}{dt} = 1 \text{ m/s}^2; \; v_{\zeta 2} = 0 \text{ m/s}^2;$$

$$t > 1220 \text{ c,} \; v_{\zeta 1} = 20 \text{ м/с,} \quad \frac{dv_{\zeta 1}}{dt} = \frac{dv_{\zeta 3}}{dt} = 0 \text{ m/s}^2; \; v_{\zeta 2} = 0 \text{ m/s}^2; \tag{5}$$

Figure 7 presents errors of the determination of the Cartesian coordinates of the object position in the northern ($\Delta\zeta_1$) and eastern ($\Delta\zeta_3$) directions as well as of the course angle ($\Delta\Psi$) under the ideal work algorithms, i.e. at the absence of the sensors errors. This simulation is considered as reference.

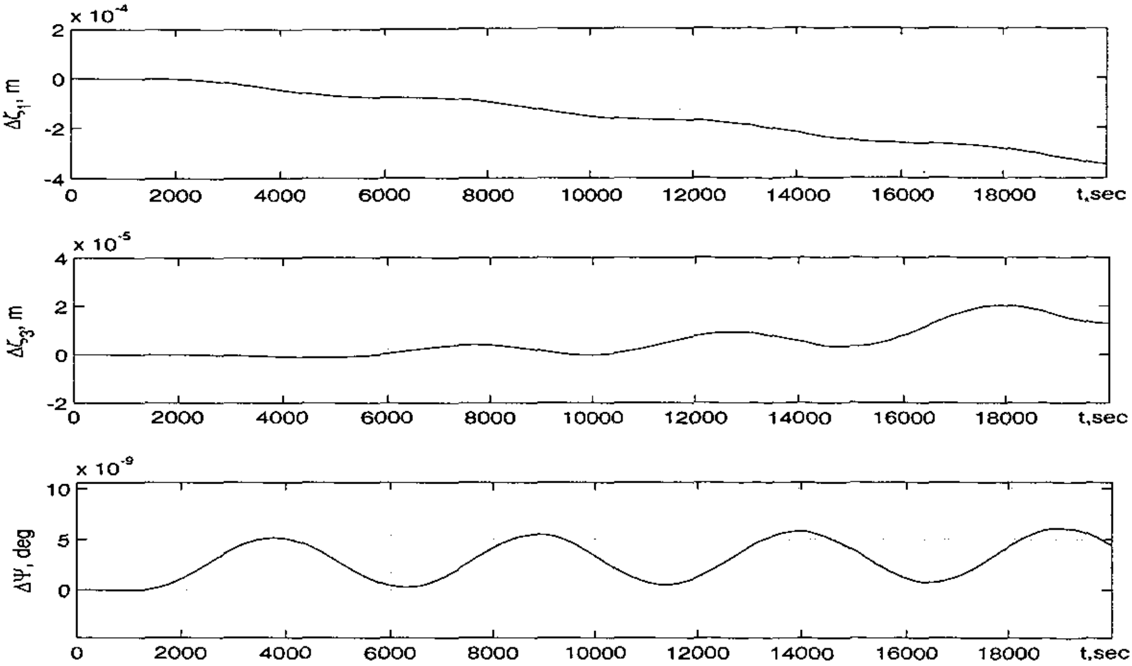

Fig. 7. SINS coordinates and course angle determination errors

It appears from the diagrams that the error are accumulated over time. It is observed oscillations with the Schuler period. By the moment of time $t$=2·10$^4$ sec the coordinate determination errors were reached values of $\Delta\zeta_1$=-3,5·10$^{-4}$ m and $\Delta\zeta_3$=1,2·10$^{-5}$ m. Yaw, roll and pitch angles errors were lesser. Travel speeds errors $\Delta V_{\zeta 1}$ and $\Delta V_{\zeta 3}$ were reached values of 0,6·10$^{-10}$, m/s and 4·10$^{-14}$, m/s. It is not difficult to see that under Coriolis acceleration and velocity errors compensation, for example by GPS signals, SINS has a high accuracy.

In order to estimate the influence of the sensors errors it were taken into account FOG errors in the form of:

- zero bias error $\Delta\omega_{xi}$ is 0,01 ($i$=1,2,3) °/hour;
- scale factor error $\delta\omega_{xi}$ is 10$^{-4}$ ($i$=1,2,3);
- FOG scale factor asymmetry $A\omega_{xi}$ is 10$^{-6}$ ($i$=1,2,3).

Initially the accelerometers errors aren't taken into account.

As a result of the mathematical modeling for mentioned conditions the following errors are obtained:

$\Delta\Psi$=0,06 °; $\Delta\zeta_1$=4200 km; $\Delta\zeta_3$=900 km; $\Delta\theta$=2·10$^{-3}$ °; $\Delta\gamma$=4·10$^{-3}$ °.

Under accounting accelerometers errors $\Delta W_{xi}$=10$^{-4}$ m/s$^2$; $\delta W_{xi}$=10$^{-4}$ ($i$=1,2,3) SINS errors came to following maximum values at the time $t$=20·10$^4$ c:

$\Delta\psi$=0,03 °; $\Delta\Psi$=0,06 °; $\Delta\theta$=7·10$^{-3}$ °; $\Delta\gamma$=1·10$^{-2}$ °; $\Delta\zeta_1$=4,2 km; $\Delta\zeta_3$=1 km.

In order to estimate the influence of gyroscopes and accelerometers noises on the SINS accuracy we need to mention the following circumstances. Noises of investigated gyroscopes have low frequency nature, and located mostly in the frequency range of 0 to 5 Hz [1,2]. Basically amplitudes of these noises are from 1 to 10, °/hour [8]. As a result of experimental it was found that the low-frequency noises have a significant influence on the accuracy of the system and probably they are detected due to gyroscopes and accelerometers static characteristics asymmetry.

Since application of sensor signals records made under experiments is limited because of number of reasons, such as the duration of recording, value and instability of sample rate, presence in sensor signals of undesirable noises and information (gyroscopes signals, for example, contain Earth angle rate and zero drift) method of "color" noises generation using the Allan variance coefficients was developed. Basis of this method is described in [9,10,11,12]. Obtained signals have nearly the same power spectral characteristics of the noise density as the experimental sensors signals.

Figure 8 provides diagrams of SINS errors of the object position coordinates determination in the northern ($\Delta\zeta_1$) and eastern ($\Delta\zeta_3$) directions and the course angle determination ($\Delta\Psi$) with taking into account sensors errors like FOG zero drift and accelerometers zero bias, scale factor error and asymmetry as well as noises which are synthesized on basis of calculated Allan variance coefficients for the experimental sensors records. Following sensors errors were taken: FOG zero bias $\Delta\omega_{xi}$ is 0,01 ($i$=1,2,3) °/hour, FOG scale factor error $\delta\omega_{xi}$ is 10$^{-4}$ ($i$=1,2,3), FOG scale factor asymmetry $A\omega_{xi}$ is 10$^{-6}$ ($i$=1,2,3), accelerometers zero bias $\Delta W_{xi}$ is 10$^{-4}$ m/s$^2$, accelerometers scale factor error $\delta W_{xi}$ is 10$^{-4}$ ($i$=1,2,3). The simulation results are SINS errors prediction and indicate that errors induced by the noise rose to following values: $\Delta\psi$=0,05 °; $\Delta\Psi$=0,2 °; $\Delta\theta=5\cdot10^{-3}$ °; $\Delta\gamma=1\cdot10^{-2}$ °; $\Delta\zeta_1$=7,5 km; $\Delta\zeta_3$=3,8 km; $t\in(0\ldots2\cdot10^4)$ sec. The sensors were taken in form of SRS-1000 and AKP-2.

The analysis of the SINS errors given as results of mathematical modeling showed that under using of several types FOG the yaw angle errors vary from 13 degree (for VG-951) to 0.35 degree (for SRS-1000). Latitude errors come to values from 825 km (for VG-951) to 31 km (for SRS-1000) for the same time. It is ascertained that mentioned above errors reduced by 2 to 3 times for SRS-1000 and by 5 to 10 times for VG-951 when noises which are mainly low-frequency were excluded from sensors signals. It is necessary to filter sensors signals in order to reduce errors induced by influence of "color" noise.

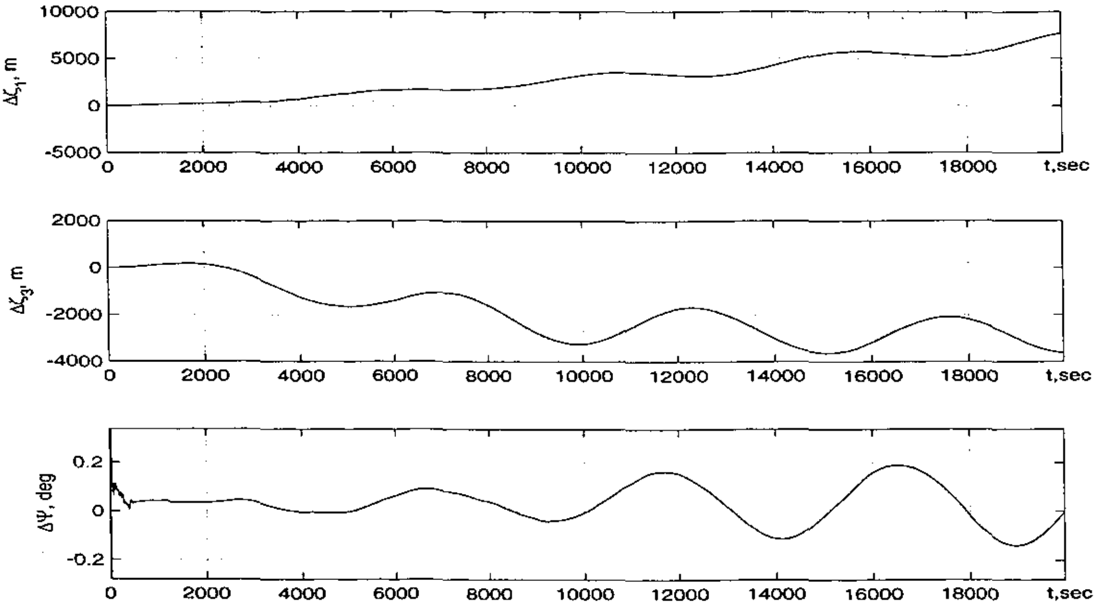

Fig. 8. SINS errors diagrams

In an effort to exposure of the separate factors influence on an SINS errors a series of the mathematical simulations was realized. It was separately estimated a number of dependences:

– influence of the gyroscopes signals noise components amplitude ("white" noise with standard deviation of 0,6; 0,1; 3 °/hour is used as a component of the gyroscopes signal noise);

– influence of the gyroscope scale factor asymmetry (assigned values are $A\omega_{xi}$=0,0001%; 0,00003%; 0,00001% ($i$=1,2,3));

– influence of accelerometers zero bias (assigned values are $\Delta W_{xi}=10^{-2}$; $10^{-3}$; $10^{-4}$ m/s$^2$ ($i$=1,2,3)).

For a more accurate estimation of the specified parameters influence the rest of the sensors errors were excluded from the calculation during each simulation. Simulation results in the form of dependence of resulting errors versus the above-mentioned parameters are presented in the figures 9 to 11.

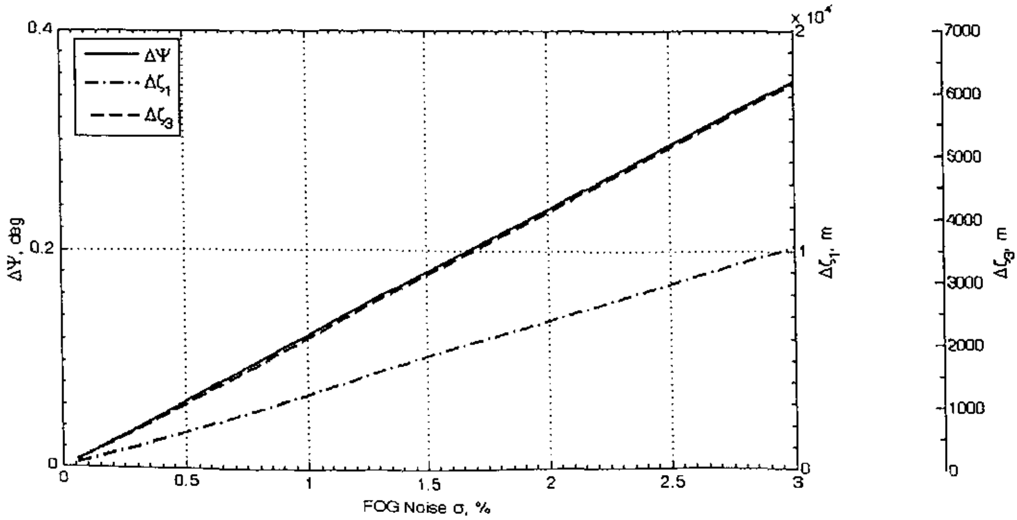

Fig. 9. Positioning and course angle errors versus gyroscope signals noise amplitude

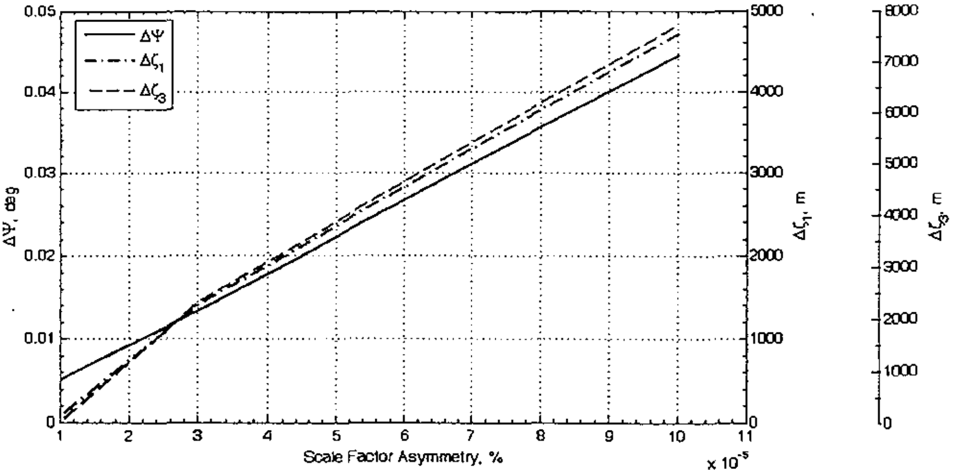

Fig. 10. Positioning and course angle errors versus value of gyroscope scale factor asymmetry

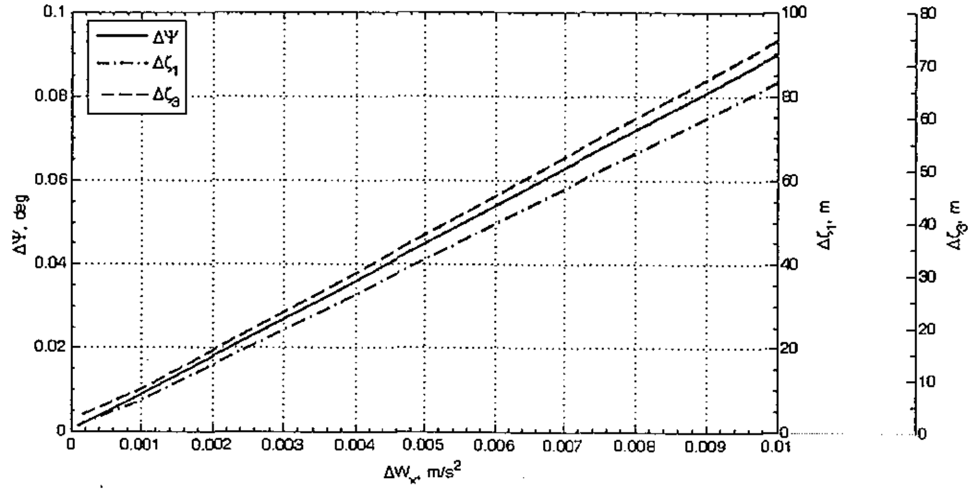

Fig. 11. Positioning and course angle errors versus от accelerometers zero bias

As it can be seen from the diagrams the gyroscopes signal noise amplitude as well as the value of scale factor asymmetry significantly effect on the positioning and course angle accuracy. With an increase of the gyroscopes output signal noise standard deviation from 0,06°/hour to 3°/hour the coordinates determination errors $\Delta\zeta_1$ and $\Delta\zeta_3$ grew from 202 m and 125 m to 10,1 km and 6,1 km respectively. The course angle determination error $\Delta\Psi$ rose from 0,007° to 0,35°. An increase of the gyroscopes scale factor asymmetry value from $10^{-5}$% to $10^{-4}$% gives a change of the coordinates determination errors $\Delta\zeta_1$ and $\Delta\zeta_3$ from 85 m and 17 m to 4,7 km and 7,7 km respectively as well as the course angle determination errors $\Delta\Psi$ grew from 0,005° to 0,045°. The value of accelerometers zero bias also affects the SINS accuracy. At that this dependency is closed to described one in [4]:

$$\Delta\Psi = \frac{\Delta\omega}{U\cos\varphi} + \frac{\Delta W}{g} tg\varphi; \tag{6}$$

here $\varphi$ latitude, $g$ is gravity acceleration. With an increase of the accelerometers zero bias value from 0,0001 m/s$^2$ до 0,01 m/s$^2$ the coordinates determination errors $\Delta\zeta_1$ and $\Delta\zeta_3$ grew from 1,5 m and 2,9 m to 83 m and 75 m respectively. The course angle determination error $\Delta\Psi$ increased from 0,001° to 0,09°.

## Conclusions

Main FOG and accelerometers errors such as zero drift and zero bias, scale factor asymmetry and intrinsic noises were experimentally determined and analyzed. At that noise estimation was made using Allan variation calculation. The approach for the SINS output parameters errors prediction with taking into account the errors of FOGs and accelerometers using the mathematical simulation of its work is described. Dependences of the SINS position and course angle determination errors versus main sensors errors were obtained using described approach.

Using the experimental data of FOG (PNSK 40-018) [13] and accelerometers (AKP-2) which have following errors: FOG zero bias $\Delta\omega_{xi}$ is 0,01 ($i$=1,2,3) °/hour, FOG scale factor error $\delta\omega_{xi}$ is $10^{-4}$ ($i$=1,2,3), FOG scale factor asymmetry $A\omega_{xi}$ is $10^{-6}$ ($i$=1,2,3), FOG noises standard deviation is 0,35 °/hour, accelerometers zero bias $\Delta W_{xi}$ is $10^{-4}$ m/s$^2$, accelerometers scale factor error $\delta W_{xi}$ is $10^{-4}$ ($i$=1,2,3), accelerometers noises standard deviation is $6\cdot10^{-4}$ m/s$^2$ next prediction estimations of SINS errors were obtained: mobile object position determination errors in the northern direction $\Delta\zeta_1$ is 7,5 km and in the eastern direction $\Delta\zeta_3$ is 3,8 km as well as the course angle error $\Delta\Psi$ is 0,2 °; during time period $t\in(0\ldots2\cdot10^4)$ sec. On the research realization the sensors misalignment angles, temperature zero drift, vibrations as well as magnetic fields influence weren't taken into account. Under such errors the position coordinates and course angle determination errors will rise.

## References

1. **IEEE** Std 952-1997. IEEE Standard Specification Format Guide and Test Procedure for Single-Axis Interferometric Fiber Optic Gyros.
2. **Буравлев А.С.** Использование вариации Аллана при исследовании шумовых характеристик волоконно-оптических гироскопов // VIII конференции молодых ученых "Навигация и управление движением" / А.С. Буравлев, Л.Г. Лисин // - СПб.: ЦНИИ "Электроприбор", 2006. (http://www.elektropribor.spb.ru/cnf/kmu8/dokl/s4-7.zip).
3. **Prince P.J. & Dorman J.R.** High order embedded Runge-Kutta formulae. J.Comp. Appl. Math. 1981, Vol. 7. pp. 67-75.
4. **Gaiffe T., Cottreau Y., Faussot N., Hardy G., Simonpietri P., Lefevre H. and Arditty H.** Highly Compact Fiber Optic Gyrocompass for applications at depths up to 3,000 meters. Underwater Technology, 2000. UT 00. Proceedings of the 2000 International Symposium on Volume , Issue , 2000 Page(s):155 - 160.
5. **Плотников П.К.** Теория работы одной разновидности бесплатформенных инерциальных систем ориентации// Гироскопия и навигация. – 1999. - №3. –С. 23-31.
6. **Анучин О.Н., Емельянцев Г.И.** Интегрированные системы ориентации и навигации для морских подвижных объектов./ О.Н. Анучин, Г.И. Емельянцев // –Л.: ЦНИИ «Электроприбор», 1999. –356 с.
7. **Ишлинский А.Ю.** Механика гироскопических систем, М: Изд-во АН СССР, 1963 –483с.
8. **Коркишко Ю.Н.** Волоконно-оптический гироскоп навигационного класса точности / Ю.Н. Коркишко, В.А. Федоров, В.Е. Прилуцкий и др. // XIV С.-Петебургская международная конференция по интегрированным навигационным системам. – СПб.: ЦНИИ «Электроприбор», 2007. – С. 141-150.
9. **Plaszczynski S.** Generating long streams of $1/f^{\alpha}$ noise. Fluctuation and Noise Letters. Vol.0.No.0 (2006) 000-000. World Scientific Publishing Company. http://arxiv.org/PS_cache/astro-ph/pdf/0510/0510081v2.pdf
10. **Keshner M.** $1/f$ noise. Proc. IEEE. Vol.70 (1982)
11. **Kasdin N.** Discrete simulation of colored noise and stochastic-processes and $1/f^{\alpha}$ power-law noise generation. Proc. IEEE. Vol.83 (1995)
12. **Haiying Hou.** Modeling Inertial Sensors Errors Using Allan Variance. Department of Geomatics Engineering. UCGE Reports. Number 20201. September 2004.
13. **Коркишко Ю.Н.** Волоконно-оптический гироскоп навигационного класса точности / Ю.Н. Коркишко, В.А. Федоров, В.Е. Прилуцкий, В.Г. Пономарев и др. // Гироскопия и навигация. – 2008. - №1. – С. 71-82.